Khandelwal Vaish Girls Institute of Technology

Internal Examination 2017 - 18

Business Mathematics and Statistics

MBA Semester I

Question Paper & Answer Key

MM: 30 Time : 02:30 hours

1. **Find the inverse of matrix A** $\begin{bmatrix} \mathbf{2} & \mathbf{2} & \mathbf{0} \\ \mathbf{2} & \mathbf{1} & \mathbf{1} \\ \mathbf{-7} & \mathbf{2} & \mathbf{-3} \end{bmatrix}$**.** (4)

Ans. $|A| = 2\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} -2\begin{vmatrix} 2 & 1 \\ -7 & -3 \end{vmatrix} +0\begin{vmatrix} 2 & 1 \\ -7 & 2 \end{vmatrix}$

$= 2(-3-2) -2 (-6+7)+0 = -12$

THE COFACTORS Cij OF A ARE,

$C_{11}=(-1)^{1+1} \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -5,$

$C_{12}=(-1)^{1+2} \begin{vmatrix} 2 & 1 \\ -7 & -3 \end{vmatrix} = -1$

$C_{13}=(-1)^{1+3} \begin{vmatrix} 2 & 1 \\ -7 & 2 \end{vmatrix} =1\ 1$

$C_{21}=(-1)^{2+1} \begin{vmatrix} 2 & 0 \\ 2 & -3 \end{vmatrix} = 6$

$C_{22}=(-1)^{2+2} \begin{vmatrix} 2 & 0 \\ -7 & -3 \end{vmatrix} = -6$

$C_{23}=(-1)^{2+3} \begin{vmatrix} 2 & 2 \\ -7 & 2 \end{vmatrix} = -18$

$C_{31}=(-1)^{3+1} \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2$

$C_{32}=(-1)^{3+2} \begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} = -2$

$C_{33}=(-1)^{3+3} \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} = -2$

THE ADJOINT OF A IS :

$$\text{Adj. A} = \begin{bmatrix} C11 & C21 & C31 \\ C12 & C22 & C32 \\ C13 & C23 & C33 \end{bmatrix} = \begin{bmatrix} -5 & 6 & 2 \\ -1 & -6 & -2 \\ 11 & -18 & -2 \end{bmatrix}$$

$$A^{-1} = \text{Adj. A}/|A| = -1/12 \begin{bmatrix} -5 & 6 & 2 \\ -1 & -6 & -2 \\ 11 & -18 & -2 \end{bmatrix} = \begin{bmatrix} 5/12 & -1/12 & -1/6 \\ 1/12 & 1/2 & 1/6 \\ -11/12 & 3/2 & 1/6 \end{bmatrix}$$

2. **Solve the system with three variables by Cramer's Rule** $\begin{cases} \mathbf{x + 2y + 3z = -5} \\ \mathbf{3x + y - 3z = 4} \\ \mathbf{-3x + 4y + 7z = -7} \end{cases}$ (4)

Ans. From the given system of linear equations, I will construct the four matrices that will be used to solve for the values of *x*, *y*, and *z*.

Use the guide above to correctly setup these special matrices.

- coefficient matrix

$$D=\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & -3 \\ -3 & 4 & 7 \end{bmatrix}$$

- X – matrix

$$D_{X}=\begin{bmatrix} -5 & 2 & 3 \\ 4 & 1 & -3 \\ -7 & 4 & 7 \end{bmatrix}$$

- Y – matrix

$$D_{Y}=\begin{bmatrix} 1 & -5 & 3 \\ 3 & 4 & -3 \\ -3 & 7 & 7 \end{bmatrix}$$

- Z – matrix

$$D_{Z}=\begin{bmatrix} 1 & 2 & -5 \\ 3 & 1 & 4 \\ -3 & 4 & -7 \end{bmatrix}$$

Next, I will solve for the determinant of each matrix. To do this, I can manually solve the determinant of each matrix on paper using the formula provided above. It can be tedious, but it's okay since good math skills are developed by doing lots of problems.

To check your work, you may use the 3×3 determinant calculator from Wolfram Alpha. The values of the determinants are listed below.

Determinants of each matrix

$$|D| = 40$$
$$|DX| = -40$$
$$|DY| = 40$$
$$|DZ = -80|$$

The final answers or solutions are easily computed or calculated once all the required determinants are found.

Solved values for *x*, *y*, and *z*

X= $|DX|/|D|$ =- 40/40 = -1

Y = $|DY|/|D|$ = 40/40 = 1

Z = $|DZ|/|D|$ = -80/40 = -2

The final answer written in point notation is $(x, y, z) = (-1, 1, -2)$

3. **There are 100 students in a class. Their marks have been tabulated in a frequency distribution having seven class intervals of equal size., the first class is 10-20 , the less then cumulative frequency of the 4th ,5th and 6th class intervals are 40, 70, 92 respectively. Calculate Mean and Median.**

   If the frequency of the second class is double of the first class but equal to the third class and frequency of the fourth class in half of the fifth class. (4)

Ans. Calculation of Frequency :- As pre information given frequency of the fifth class is (70-40)=30; frequency of 6th class is (92-70)=22 ; frequency of 7th class is (100-92) =8 ; now the frequency of fourth is half of the fifth class i.e., 30*1/2=15. HENCE THEW TOTAL FREUENCIES OF FIRST three class intervals are (40-15)= 25. Now this 25 is in ration of 1:2:2 for first three class interval to the frequency of first three classes are (25=1:2:2)5, 10 and 10 respectively.

Calculation of Mean and Median

Group	*f*	M.V (X)	(X-A) A=45 (dx)	dx'	fd'x	cf
10-20	5	15	-30	-2	-15	5
20-30	10	25	-20	-2	-20	15
30-40	10	35	-10	-1	-10	25
40-50	15	45	0	0	0	40
50-60	30	55	10	1	30	70
60-70	22	65	20	2	44	92
70-80	8	75	30	3	24	100

N=100 ∑fd'x = +53

<u>Mean:</u> X = A+ ∑fd'x\N *i

= 45+ 53\100 *10

= 45 + 5.3+ 50.3 Ans.

Median: m= size of 100\2 = 50th item.

So median group is (50-60)

M = L + I\F (m-c_o)

= 50 + 10\30 (50-40)

=50+10\30 *10 = 50 +3.3 53.3 Ans.

4. **Find the value of the correlation coefficient from the following table:** (3)

Subject	**Age x**	**Glucose level**

		y
1	43	99
2	21	65
3	25	79
4	42	75
5	57	87
6	59	81

Ans. Step 1: Multiply x and y together to fill the xy column. For example, row 1 would be 43 × 99 = 4,257.

Subject	Age X	Glucose Level Y	XY	X^2	Y^2
1	43	99	4257		
2	21	65	1365		
3	25	79	1975		
4	42	75	3150		
5	57	87	4959		
6	59	81	4779		

Step 2: Take the square of the numbers in the x column, and put the result in the x^2 column.

Subject	Age X	Glucose Level Y	XY	X^2	Y^2
1	43	99	4257	1849	
2	21	65	1365	441	
3	25	79	1975	625	
4	42	75	3150	1764	
5	57	87	4959	3249	
6	59	81	4779	3481	

Step 3: Take the square of the numbers in the y column, and put the result in the y^2 column.

Subject	Age X	Glucose Level Y	XY	X^2	Y^2
1	43	99	4257	1849	9801
2	21	65	1365	441	4225
3	25	79	1975	625	6241
4	42	75	3150	1764	5625
5	57	87	4959	3249	7569
6	59	81	4779	3481	6561

Step 4: Add up all of the numbers in the columns and put the result at the bottom of the column. The Greek letter sigma (Σ) is a short way of saying "sum of."

Subject	Age X	Glucose Level Y	XY	X^2	Y^2
1	43	99	4257	1849	9801
2	21	65	1365	441	4225
3	25	79	1975	625	6241
4	42	75	3150	1764	5625
5	57	87	4959	3249	7569
6	59	81	4779	3481	6561
Σ	247	486	20485	11409	40022

Step 6: Use the following correlation coefficient formula.

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

From our table:

$\Sigma X = 247$
$\Sigma Y = 486$
$\Sigma XY = 20{,}485$
$\Sigma X^2 = 11{,}409$
$\Sigma Y^2 = 40{,}022$
n is the sample size, in our case = 6

The correlation coefficient

$r = 6(20{,}485) – (247 \times 486) / [\sqrt{[[6(11{,}409) – (247^2)] \times [6(40{,}022) – 486^2]]}]$
$= 0.5298$

The range of the correlation coefficient is from -1 to 1. Our result is 0.5298 or 52.98%, which means the variables have a moderate positive correlation.

5. **Determine the regression equation by using the regression slope coefficient and intercept value as shown in the regression table given below.** (4)

X Values	Y Values
55	52
60	54
65	56
70	58
80	62

For the given data set of data, solve the regression slope and intercept values.

Ans. Let us count the number of values.

N = 5

Determine the values for XY, X^2

X Value	Y Value	X*Y	X*X
55	52	2860	3025
60	54	3240	3600
65	56	3640	4225
70	58	4060	4900
80	62	4960	6400

Determine the following values $\sum X$, $\sum Y$, $\sum XY$, $\sum X^2$.

$\sum X = 330$

$\sum Y=282$

$\sum XY=18760$

$\sum X^2=22150$

Substitute values in the slope formula

Slope (b) = $\{N\sum XY-(\sum X)(\sum Y)\} / \{N\sum X^2-(\sum X)^{2\}}$

$= \{(5)\times(18760)-(330)\times(282)\} / \{(5)\times(22150)-(330)^2\}$

$b = 0.4$

Substitute the values in the intercept formula given.

Intercept (a) = $\sum Y-b(\sum X)N$

$= 282 - 0.4\ (330)\ 5$

$a=30$

Substitute the Regression coefficient value and intercept value in the regression equation Regression Equation (y) = a + bx

$y = 30 + 0.4x$

6. **From the following data calculate price index number for 2010 with 2000 as base by Fisher's ideal index method.** (4)

Ans. Calculation of the Price Indices

Commodities	**Base Year (0)**		**Current Year (1)**		p_1q_0	p_0q_0	p_1q_1	p_0q_1
	p_0	q_0	p_1	q_1				
A	20	8	40	6	320	160	240	120
B	50	10	60	5	600	500	300	250

C	40	15	50	15	750	600	750	600
D	20	20	20	25	400	400	500	500
					$\sum p_1q_0$ = 2070	**$\sum p_0q_0$ = 1660**	**$\sum p_1q_1$ = 1790**	**$\sum p_0q_1$ = 1470**

Fisher's Index Number

$$P01=\sqrt{\sum P1Q0 \backslash \sum P0Q0 * \sum P1Q1 \backslash \sum P0Q1} * 100$$

$$=\sqrt{2070\backslash 1660 * 1790\backslash 1470} * 100$$

$$=\sqrt{207\backslash 166 * 179\backslash 147} * 100$$

=123.24 Answer

7. **Two dice are rolled, find the probability that the sum is** (3)
   a) equal to 1
   b) equal to 4
   c) less than 13

Ans. The sample space S of two dice is shown below:

S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) }

A. Let E be the event "sum equal to 1". There are no outcomes which correspond to a sum equal to 1, hence

P(E) = n(E) / n(S)
= 0 / 36
= 0

B. Three possible outcomes give a sum equal to 4:

E = {(1,3),(2,2),(3,1)}, hence
P(E) = n(E) / n(S)
= 3 / 36
= 1 / 12

C. All possible outcomes, E = S, give a sum less than 13, hence.

P(E) = n(E) / n(S)
= 36 / 36
= 1

8. **Fit a Poisson distribution to the following data with respect to the number of red blood corpuscles (X). Find expected values also.** (4)

X	0	1	2	3	4	5	6	7	8
f	162	193	115	83	44	24	19	8	2

Ans. P(X=x)= $\boldsymbol{e}^{-\boldsymbol{\lambda}}\ \boldsymbol{x}!\ \boldsymbol{\lambda}^{x}$ = 0

X = 0, 1, 2, 3 ……

Mean = Variance = $\boldsymbol{\lambda}$

Mean = $\boldsymbol{X}$ = $\boldsymbol{\lambda}$

Where $\boldsymbol{X} = \sum \boldsymbol{f}\boldsymbol{x} / \sum f$

Calculations:- First we have to calculate about mean.

X	**f**	**fX**
0	162	0
1	193	193
2	115	230
3	83	249
4	44	176
5	24	120
6	19	114
7	8	56
8	2	16
	$\sum f = 650$	$\sum fX = 1154$

Here, Mean = $\boldsymbol{X} = \boldsymbol{\lambda} = \mathbf{1154}/\ \mathbf{650}$ = 1.775

X	f	Probability P(X= x)= $\boldsymbol{e}^{-\boldsymbol{\lambda}}\ \boldsymbol{x}!\ /\ \boldsymbol{\lambda}^{\boldsymbol{x}}$;	Expected Frequency N.P(X= x)
0	162	0.169483	110.16
1	193	0.300833	195.54
2	115	0.266989	173.54
3	83	0.157969	102.68
4	44	0.070099	45.56
5	24	0.024885	16.18
6	19	0.007362	4.79
7	8	0.001867	1.21
8	2	0.000414	0.27
	650		650

P(X = x) = $\boldsymbol{e}^{-\mathbf{1.775}} * (\mathbf{1.775})^{x} / \boldsymbol{x}!$

for x=0, 1, 2…8

X	0	1	2	3	4	5	6	7	8
f_0	162	193	115	83	44	24	19	8	2
f_1	129	208	169	91	37	12	3	1	0